Egomotion from optical flow with an uncalibrated camera

Michael J. Brooks

Department of Computer Science, University of Adelaide
Adelaide, SA 5005, Australia

Luis Baumela

Departamento de Inteligencia Artificial, Universidad Politécnica de Madrid
Campus de Montegancedo s/n, 28660 Boadilla del Monte, Madrid, Spain

Wojciech Chojnacki

Department of Computer Science, University of Adelaide
Adelaide, SA 5005, Australia

ABSTRACT

The problem of automatically determining an uncalibrated camera's motion through space solely from its view of the static surroundings has only recently received attention. In this work, we present a new direct method for computing camera egomotion from optical flow data in the particular case of a camera having unknown and possibly varying focal length. Here, egomotion refers to motion that is expressed with respect to the camera's local frame of reference. No restrictions are placed on the nature of the camera's motion other than that its translational and rotational components vary smoothly. Essential to the approach is the derivation of a differential form of the *time-dependent epipolar equation* for a single moving camera. The method requires that two special matrices be computed from optical flow data. Closed-form expressions, presented in terms of the entries of the two matrices, are then given for the egomotion parameters, the focal length and its derivative. This self-calibration process constitutes an essential prerequisite to obtaining a reconstruction of the viewed scene from optical flow.

Keywords: Egomotion, uncalibrated camera, self-calibration, epipolar equation.

1 INTRODUCTION

Considerable progress has been made in recent years in carrying out stereo vision with a partially uncalibrated setup. A major problem has been: how may we reconstruct a 3D description of a scene viewed by a pair of cameras whose intrinsic characteristics are not fully known, and whose relative orientation is unknown? Remarkably, such a description can sometimes be obtained solely by consideration of corresponding image points. Along the way, it is necessary to carry out a process of *self-calibration*, whereby the unknown imaging parameters are determined.

A single scene point projected onto two image planes gives rise to a pair of corresponding (or homologous) points. These points may arise in two images obtained from a static pair of cameras, or in two images obtained at different times by a moving camera. In the latter context, as the time difference tends to zero, we may think of corresponding points as tending to optical flow, wherein instantaneous velocities of various image points are recorded.

Analogously to carrying out self-calibration for a stereo vision setup from corresponding points, our aim in this work is to carry out self-calibration for a single moving camera from optical flow—this process being an essential prerequisite to reconstruction. Self-calibration in this latter context amounts to automatically determining the camera motion through space, as well as some of the camera's intrinsic parameters. In order to fulfil our aim, we shall start with the constraint that underpins stereo vision, and modify it so as to obtain a differential form suitable for motion vision.

The *epipolar equation* in stereo vision may be expressed as

$$\mathbf{m}^T \mathbf{F} \mathbf{m}' = 0, \tag{1}$$

where $\mathbf{m}$ and $\mathbf{m}'$ are corresponding points in the images obtained by left and right cameras, expressed in homogeneous coordinates (with each third coordinate equal to 1), and $\mathbf{F}$ is the *fundamental matrix* influenced by both extrinsic and intrinsic imaging factors, henceforth termed the *key parameters* [8]. (Note that a slightly non-standard notation is used here, as described in Appendix A.) Given sufficiently many corresponding points, it is sometimes possible, via a process of *self-calibration*, to determine various of the key parameters [3, 9].

In this work, we introduce into (1) a dependency on *time*, derive a corresponding differential equation, and exploit it to carry out self-calibration (determining camera motion and intrinsic parameter values) using only optical flow information. Part of our work may be seen as a recasting of the research of Viéville and Faugeras [10] into an analytical framework. For related work dealing with *egomotion* of a *calibrated* camera, see for example [4–7].

2 DIFFERENTIAL FORMS OF THE TIME-DEPENDENT EPIPOLAR EQUATION

The starting point of our analysis is the observation that, in contrast with the fundamental matrix associated with a pair of cameras, the fundamental matrix associated with an image pair obtained from a single camera is dependent upon *two* times. For a pair of images obtained from a single camera at times t_1 and t_2, denote by $\mathbf{F}(t_1, t_2)$ the fundamental matrix associated with this pair, and denote by $\mathbf{m}(t_1)$ and $\mathbf{m}(t_2)$ the images of a fixed 3D point in space generated at t_1 and t_2, respectively. The epipolar equation then becomes

$$\mathbf{m}^T(t_1)\mathbf{F}(t_1, t_2)\mathbf{m}(t_2) = 0. \tag{2}$$

This we may term *the time-dependent epipolar equation for a single camera.* To our knowledge, the epipolar equation in this simple but valuable form has not previously appeared in the literature. Our first goal is to obtain differential forms of this equation, in which changes in image features (optical flow) are related to changes in the parameters embedded within the fundamental matrix (egomotion).

A critical factor at this stage is to consider precisely what form differentiation with respect to time should take, given that there are two times involved. For a vector or matrix entity $\mathbf{X}(t_1, t_2)$ dependent on two times, it proves appropriate to differentiate $\mathbf{X}(t_1, t_2)$ partially with respect to t_2 at $(t_1, t_2) = (t, t)$, t being an arbitrarily fixed time instant. We denote the resulting derivative as $\overset{\circ}{\mathbf{X}}(t)$.

Single differentiation of (2) along these lines then yields

$$\mathbf{m}^T(t)\overset{\circ}{\mathbf{F}}(t)\mathbf{m}(t) = 0,$$

which we term *the first differential form of the epipolar equation.* Similarly, differentiating twice yields

$$\mathbf{m}^T(t)\overset{\circ\circ}{\mathbf{F}}(t)\mathbf{m}(t) + 2\mathbf{m}^T(t)\overset{\circ}{\mathbf{F}}(t)\dot{\mathbf{m}}(t) = 0,$$

termed *the second differential form of the epipolar equation.* Here $\mathbf{m}(t)$ and $\dot{\mathbf{m}}(t)$ constitute optical flow, with the dot denoting standard differentiation with respect to time. Note that this equation contains both location and velocity of an image point but not its acceleration, $\ddot{\mathbf{m}}(t)$ having fallen away in the derivation. Full details of this and subsequent derivations are to be found in Brooks et al. [1].

3 ELABORATING THE SECOND DIFFERENTIAL FORM

We may now elaborate the second differential form by noting that the fundamental matrix $\mathbf{F}(t_1, t_2)$ for a single camera can be expressed as

$$\mathbf{F}(t_1, t_2) = \mathbf{A}^T(t_1)\mathbf{T}(t_1, t_2)\mathbf{R}(t_1, t_2)\mathbf{A}(t_2),$$

where the matrix $\mathbf{A}(t)$ describes the intrinsic parameters of the camera at instant t, and the matrices $\mathbf{T}(t_1, t_2)$ and $\mathbf{R}(t_1, t_2)$ embody the translational and rotational components, respectively, of the camera's movement from its position at time t_1 to its position at time t_2. The intrinsic parameters within $\mathbf{A}(t)$ are assumed to vary continuously with time. The translation matrix $\mathbf{T}(t_1, t_2)$ takes the form

$$\mathbf{T}(t_1, t_2) = \begin{pmatrix} 0 & -\mathsf{z}(t_1, t_2) & \mathsf{y}(t_1, t_2) \\ \mathsf{z}(t_1, t_2) & 0 & -\mathsf{x}(t_1, t_2) \\ -\mathsf{y}(t_1, t_2) & \mathsf{x}(t_1, t_2) & 0 \end{pmatrix},$$

where $(\mathsf{x}(t_1, t_2), \mathsf{y}(t_1, t_2), \mathsf{z}(t_1, t_2))^T$ is the baseline vector that connects the optical centres of the camera at times t_1 and t_2. The rotation matrix $\mathbf{R}(t_1, t_2)$ is given by

$$\mathbf{R}(t_1, t_2) = \mathbf{R}_1(\alpha)\mathbf{R}_2(\beta)\mathbf{R}_3(\gamma),$$

where the component matrices

$$\mathbf{R}_1(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix},$$

$$\mathbf{R}_2(\beta) = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix},$$

$$\mathbf{R}_3(\gamma) = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

correspond to counter-clockwise rotations about the camera-centered coordinate axes x, y, and z by the angles α, β and γ, respectively. For convenience, the dependency of α, β and γ upon (t_1, t_2) is left implicit.

A straightforward computation reveals that

$$\overset{\circ}{\mathbf{T}}(t) = \begin{pmatrix} 0 & -\overset{\circ}{\mathsf{z}}(t) & \overset{\circ}{\mathsf{y}}(t) \\ \overset{\circ}{\mathsf{z}}(t) & 0 & -\overset{\circ}{\mathsf{x}}(t) \\ -\overset{\circ}{\mathsf{y}}(t) & \overset{\circ}{\mathsf{x}}(t) & 0 \end{pmatrix},$$

$$\overset{\circ}{\mathbf{R}}(t) = \begin{pmatrix} 0 & \overset{\circ}{\gamma}(t) & -\overset{\circ}{\beta}(t) \\ -\overset{\circ}{\gamma}(t) & 0 & \overset{\circ}{\alpha}(t) \\ \overset{\circ}{\beta}(t) & -\overset{\circ}{\alpha}(t) & 0 \end{pmatrix}.$$

The vectors $(\overset{\circ}{\mathsf{x}}(t), \overset{\circ}{\mathsf{y}}(t), \overset{\circ}{\mathsf{z}}(t))^T$ and $(\overset{\circ}{\alpha}(t), \overset{\circ}{\beta}(t), \overset{\circ}{\gamma}(t))^T$ associated with $\overset{\circ}{\mathbf{T}}(t)$ and $\overset{\circ}{\mathbf{R}}(t)$ capture the instantaneous *translational* and *angular velocities* of camera egomotion, respectively.

By appropriate substitution, the second differential form expands to:

$$\mathbf{m}^T \mathbf{A}^T \overset{\circ}{\mathbf{T}} \overset{\circ}{\mathbf{R}} \mathbf{A} \mathbf{m} + \mathbf{m}^T \mathbf{A}^T \overset{\circ}{\mathbf{T}} \dot{\mathbf{A}} \mathbf{m} + \mathbf{m}^T \mathbf{A}^T \overset{\circ}{\mathbf{T}} \mathbf{A} \dot{\mathbf{m}} = 0.$$

Observe that even though this equation incorporates the first and second derivatives of the fundamental matrix, no second derivatives of its component matrices survive the elaboration. Note also that $\mathbf{A}$ is differentiated in the standard way since it is a simple function of time.

Letting $\mathbf{B} = \dot{\mathbf{A}} \mathbf{A}^{-1}$, set

$$\begin{aligned} \mathbf{C} &= \frac{1}{2} \mathbf{A}^T (\overset{\circ}{\mathbf{T}} \overset{\circ}{\mathbf{R}} + \overset{\circ}{\mathbf{R}} \overset{\circ}{\mathbf{T}} + \overset{\circ}{\mathbf{T}} \mathbf{B} - \mathbf{B}^T \overset{\circ}{\mathbf{T}}) \mathbf{A}, \\ \mathbf{V} &= \mathbf{A}^T \overset{\circ}{\mathbf{T}} \mathbf{A}. \end{aligned} \tag{3}$$

Direct verification shows that the second differential form can be expressed as

$$\mathbf{m}^T \mathbf{C} \mathbf{m} + \mathbf{m}^T \mathbf{V} \dot{\mathbf{m}} = 0. \tag{4}$$

This equation forms the basis for our method of self-calibration. A constraint similar to that of (4), termed *the first-order expansion of the fundamental motion equation*, is derived by Viéville and Faugeras [10]. In contrast with the above, however, it takes the form of an approximation rather than a strict equality.

The matrix $\mathbf{V}$ is antisymmetric, and so, for some vector $\mathbf{v} = (v_1, v_2, v_3)^T$, it can be written as

$$\mathbf{V} = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix}.$$

The matrix $\mathbf{C}$ is symmetric, and hence it is uniquely determined by the entries $c_{11}, c_{12}, c_{13}, c_{22}, c_{23}, c_{33}$. Denote by $\boldsymbol{\pi}(\mathbf{C}, \mathbf{V})$ the composite ratio

$$\boldsymbol{\pi}(\mathbf{C}, \mathbf{V}) = (c_{11} : c_{12} : c_{13} : c_{22} : c_{23} : c_{33} : v_1 : v_2 : v_3).$$

Note that $\boldsymbol{\pi}(\mathbf{C}, \mathbf{V})$ captures the essential entries of $\mathbf{C}$ and $\mathbf{V}$ to within a common scalar factor.

The importance of (4) stems from the fact that it can be used to determine $\boldsymbol{\pi}(\mathbf{C}, \mathbf{V})$ directly from image data. Indeed, if, at any given instant t, we supply sufficiently many independent $\mathbf{m}_i(t)$ and $\dot{\mathbf{m}}_i(t)$, then $\mathbf{C}(t)$ and $\mathbf{V}(t)$ can be determined, up to a common scalar factor, from the following system of equations linear in the entries of $\mathbf{C}(t)$ and $\mathbf{V}(t)$:

$$\mathbf{m}_i(t)^T \mathbf{C}(t) \mathbf{m}_i(t) + \mathbf{m}_i(t)^T \mathbf{V}(t) \dot{\mathbf{m}}_i(t) = 0.$$

4 SPECIAL CASE: FREE FOCAL LENGTH

We now introduce some camera parameters into our analysis. Let a *free* parameter be one that is unknown and which may vary continuously with time. Assume that the focal length is free, pixels are square, and the principal point and other intrinsic parameters are fixed and known. In this situation, for each time instant t, $\mathbf{A}(t)$ is given by

$$\mathbf{A}(t) = \begin{pmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ 0 & 0 & -f(t) \end{pmatrix},$$

where u_0 and v_0 are the coordinates of the known principal point, and $f(t)$ is the focal length at time t. It emerges that, with the adoption of this form of $\mathbf{A}$, we may express $(\mathring{\alpha}, \mathring{\beta}, \mathring{\gamma})$, $(\mathring{\mathsf{x}} : \mathring{\mathsf{y}} : \mathring{\mathsf{z}})$, f and $\dot{f}$ in terms of $\boldsymbol{\pi}(\mathbf{C}, \mathbf{V})$. This we now outline.

We first make a reduction to the case $u_0 = v_0 = 0$. To this end, we represent $\mathbf{A}$ as

$$\mathbf{A} = \mathbf{A}_1 \mathbf{A}_2,$$

where

$$\mathbf{A}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -f \end{pmatrix}, \qquad \mathbf{A}_2 = \begin{pmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ 0 & 0 & 1 \end{pmatrix},$$

and let

$$\mathbf{C}_1 = (\mathbf{A}_2^{-1})^T \mathbf{C} \mathbf{A}_2^{-1}, \qquad \mathbf{V}_1 = (\mathbf{A}_2^{-1})^T \mathbf{V} \mathbf{A}_2^{-1}.$$

As it turns out, passing to $\mathbf{A}_1$, $\mathbf{C}_1$ and $\mathbf{V}_1$ in lieu of $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{V}$, respectively, amounts to assuming that $u_0 = v_0 = 0$.

Let

$$\delta_1 = \frac{\mathring{\alpha}}{f}, \qquad \delta_2 = \frac{\mathring{\beta}}{f}, \qquad \delta_3 = \mathring{\gamma}, \qquad \delta_4 = f^2, \qquad \delta_5 = \frac{\dot{f}}{f}.$$

Detailed calculation shows that δ_1, δ_2, and δ_3 satisfy

$$\begin{aligned} \delta_1 &= \frac{2c_{12}v_2 - (c_{22} - c_{11})v_1}{v_1^2 + v_2^2}, \\ \delta_2 &= \frac{2c_{12}v_1 + (c_{22} - c_{11})v_2}{v_1^2 + v_2^2}, \\ \delta_3 &= \frac{c_{11}v_1^2 + 2c_{12}v_1v_2 + c_{22}v_2^2}{v_3(v_1^2 + v_2^2)}. \end{aligned} \tag{5}$$

The expressions on the right-hand side are independent of the scale of $\mathbf{C}$ and $\mathbf{V}$. The above equations can therefore be regarded as formulae for δ_1, δ_2, and δ_3 in terms of $\boldsymbol{\pi}(\mathbf{C}, \mathbf{V})$.

Let

$$d_1 = 2c_{13} + v_1\delta_3, \qquad d_2 = 2c_{23} + v_2\delta_3, \qquad d_3 = c_{33}.$$

Further calculation shows that

$$\begin{aligned} \delta_4 &= \frac{1}{\Gamma}\left(v_1v_3d_1 + v_2v_3d_2 - (v_1^2 + v_2^2)d_3\right), \\ \delta_5 &= \frac{1}{\Gamma}\big(((v_1v_2\delta_1 + (v_2^2 + v_3^2)\delta_2)d_1 - ((v_1^2 + v_3^2)\delta_1 + v_1v_2\delta_2)d_2 \\ &\qquad + (v_2v_3\delta_1 - v_1v_3\delta_2)d_3\big), \end{aligned} \tag{6}$$

where $\Gamma = (v_1^2 + v_2^2 + v_3^2)(v_1\delta_1 + v_2\delta_2)$. Again the expressions on the right-hand side are independent of the scale of $\mathbf{C}$ and $\mathbf{V}$, and so the above equations can be regarded as formulae for δ_4 and δ_5 in terms of $\boldsymbol{\pi}(\mathbf{C}, \mathbf{V})$.

Now, with formulae (5) and (6) at hand, the parameters $\mathring{\alpha}$, $\mathring{\beta}$, $\mathring{\gamma}$, f and $\dot{f}$ are given by

$$\mathring{\alpha} = \delta_1\sqrt{\delta_4}, \qquad \mathring{\beta} = \delta_2\sqrt{\delta_4}, \qquad \mathring{\gamma} = \delta_3, \qquad f = \sqrt{\delta_4}, \qquad \dot{f} = \delta_5\sqrt{\delta_4},$$

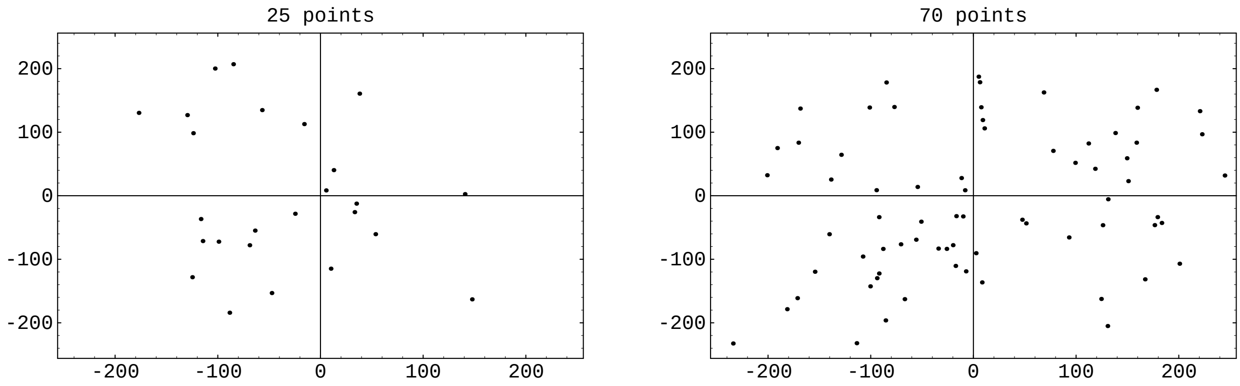

Figure 1: 3D points projected onto the image plane.

and the direction of the translational velocity is given by

$$(\mathring{\mathsf{x}} : \mathring{\mathsf{y}} : \mathring{\mathsf{z}}) = (-v_1 : -v_2 : f v_3).$$

We now summarise our procedure for carrying out self-calibration from optical flow. Assuming that a camera moves smoothly but arbitrarily through space, and has unknown and possibly varying focal length, the following steps are undertaken:

- Obtain optical flow data ($\mathbf{m}$ and $\dot{\mathbf{m}}$) for some given instant.
- Estimate the 3×3 matrices $\mathbf{C}$ and $\mathbf{V}$ from the optical flow information using standard numerical techniques.
- Compute the focal length and its derivative using closed-form expressions in the entries of $\mathbf{C}$ and $\mathbf{V}$.
- Determine the camera egomotion (up to a constant factor in translation speed), again using closed-form expressions in the entries of $\mathbf{C}$ and $\mathbf{V}$.

5 EXPERIMENTAL RESULTS

In this section, we present results of two experiments. In order to enable comparison with ground truth, the experiments were conducted with the aid of synthetic data.

Two sets of 3D points were generated, one containing 25 and the other containing 70 points. Points in each set were uniformly distributed over a 2 metre cube located 3 metres from the camera. These data points were projected onto images of size 512×512 (square) pixels, assuming a focal length of 384 pixels. Figure 1 depicts images of the two sets of points. It was assumed that $(u_0, v_0) = (0, 0)$.

During the simulation, the camera's translational velocity $(\mathring{x}, \mathring{y}, \mathring{z})$, rotational velocity $(\mathring{\alpha}, \mathring{\beta}, \mathring{\gamma})$, and velocity $\dot{f}$ of focal length were controlled. Optical flow was synthesised before being perturbed with noise. Image velocity $\dot{\mathbf{m}}$ was perturbed by adding a two-dimensional random variable with components formed by two independent copies

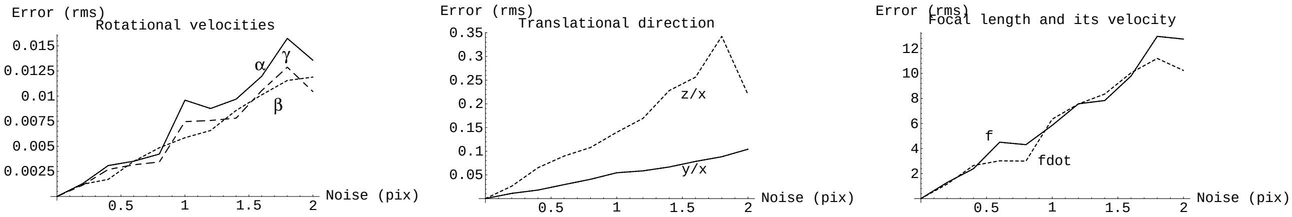

Figure 2: Errors in key parameters versus optical flow noise, with 25 points.

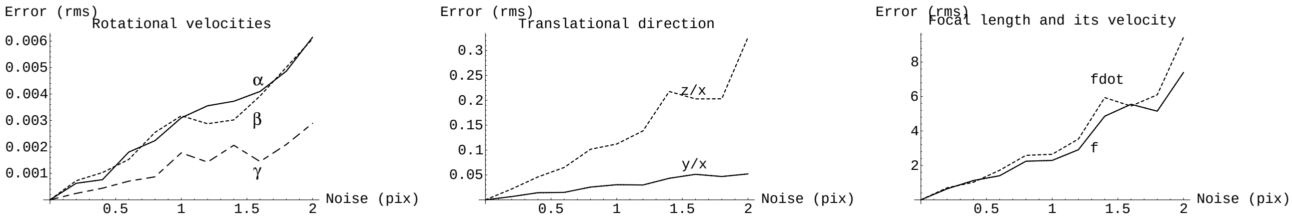

Figure 3: Errors in key parameters versus optical flow noise, with 70 points.

of a single one-dimensional random variable ν. In our experiments ν was taken to be uniformly distributed over the interval $[-2, 2]$, in pixel units.

Once all velocities were perturbed, the entries of $\mathbf{C}$ and $\mathbf{V}$ were computed via singular value decomposition, and from these the motion parameters and focal length information were estimated. This was repeated 25 times, with variation arising as a result of differing noise contamination. Finally, the root-mean-square (rms) error of each of the estimated parameters was computed.

First considered was the impact of optical flow noise on the estimation of the motion parameters, in each of the cases of 25-point flow and 70-point flow. The parameters with which the synthetic data were generated were $\mathring{\alpha} = 0.2$, $\mathring{\beta} = 0.1$, $\mathring{\gamma} = 0.4$, $\mathring{\mathsf{x}} = 0.3$, $\mathring{\mathsf{y}} = 0.3$, $\mathring{\mathsf{z}} = 0.5$, $f = 384$, and $\dot{f} = 1$. The results of the tests are shown in Figures 2 and 3. The average length of a flow vector was 105 pixels, with the maximum and minimum velocities being equal to 190 and 15 pixels.

Next we considered the impact of a perturbation in the location of the image principal point. Figure 4 shows plots of a perturbation in this location of up to 10 pixels versus errors in various of the key parameters.

Based on these experiments, we conclude that our self-calibration technique is reasonably well-behaved in the presence of noise, in the sense that:

(i) key parameter estimation error grows approximately linearly with the strength of optical flow contamination;

(ii) key parameter estimation error grows approximately linearly with error in the principal point location;

(iii) rms error in the estimation of key parameters tends to decrease as the number of data points grows.

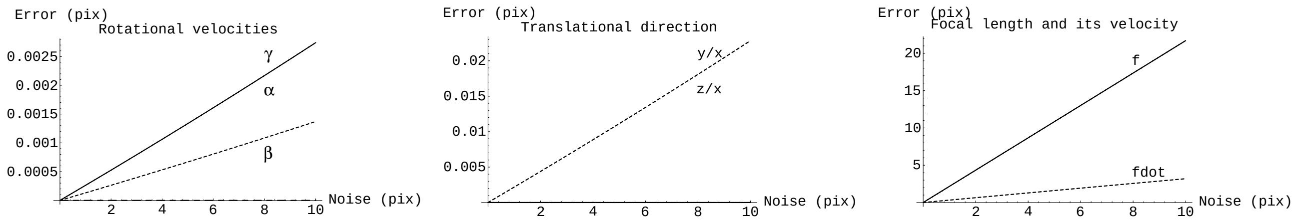

Figure 4: Errors in key parameters versus error in principal point location.

Nevertheless, if we consider the signal to noise ratio, that is the relationship between the average velocity magnitude (105 pixels) and the contaminating noise ($[-2, +2]$ pixels), it is evident that, as with previous methods, our technique remains relatively sensitive to noise in optical flow.

6 ACKNOWLEDGEMENTS

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A NOTATION SEMANTICS

Our notation differs from the standard notation of Faugeras et al. [3] (henceforth termed the Faugeras notation). Symbols $\mathbf{F}$, $\mathbf{T}$, $\mathbf{R}$ and $\mathbf{A}$ denote in this work the fundamental, translation, rotation and intrinsic-parameter matrices, respectively. Let the corresponding matrices in Faugeras notation be denoted F, T, R and A. Herein, the epipolar equation has the form $\mathbf{m}^T \mathbf{F} \mathbf{m}' = 0$, where $\mathbf{F} = \mathbf{A}^T \mathbf{T} \mathbf{R} \mathbf{A}'$. This contrasts with Faugeras notation, where $\mathbf{m}'^T F \mathbf{m} = 0$, and $F = A'^{-T} T R A^{-1}$. The full list of notational relationships is as follows:

$$\begin{aligned} \mathbf{F} &= \sqrt{\det(A)\ \det(A')} F^T, \\ \mathbf{T} &= -R^T T R, \\ \mathbf{R} &= R^T, \\ \mathbf{A} &= -\sqrt{\det(A)} A^{-1}. \end{aligned}$$

See [2] for further discussion.