ESTIMATING SUN DIRECTION FROM A SINGLE IMAGE

WOJCIECH CHOJNACKI and MICHAEL J. BROOKS
Department of Computer Science, University of Adelaide
GPO Box 498, Adelaide, SA 5001, Australia

and

DANIEL GIBBINS
School of Information Science and Technology
Flinders University of South Australia, Australia
GPO Box 2100, Adelaide, SA 5001

ABSTRACT

This paper focusses on the pioneering and influential method of Pentland which automatically estimates the direction of the "sun" from a single image. We show that, under the assumptions used in the derivation of the method, the estimate of source direction is incorrect. Specifically, we show that an image-based expression used in calculating source direction diverges to infinity as the density of image points is increased, and that, as a result, the formula involving this expression is incorrect. Supporting experimental evidence is given. We propose an alternative source estimator which is free of these drawbacks.

1. Introduction

Much work has been done to develop automated techniques for estimating the direction of the light source, or "sun", from a single image of a scene[4,6]. This work has been motivated by the desire to enable various existing shape-from-shading algorithms[3] to operate with less prerequisite information. Estimating light source direction given only a single image is a highly ill-posed problem. As a result, techniques that attempt to solve this problem need to make strong assumptions about both the shape and the reflectance properties of the object depicted. For example, many estimators assume that the image depicts a Lambertian surface that is, at least locally, spherical in shape. Inevitably, the performance of an estimator will depend in good measure on the extent to which the underlying assumptions are satisfied by the object in view. Not always will the assumptions well approximate the given circumstances.

The first and most influential attempt to develop a technique for automatic recovery of light source direction is due to Pentland[5]. In Pentland's estimator, light source direction is couched in terms of the directional derivatives of image irradiance. In this paper, we show that Pentland's method contains a flaw. When the method is implemented, this results in an estimator for light source direction that is dependent on the density of image points, or image resolution. The method also gives inaccurate results even in the presence of data that are perfectly consistent with the assumptions under which the method is derived.

In attempting to remedy Pentland's technique, we have developed an estimator that we term the disc method. This method retains the image-based expressions of Pentland, but combines them in such a way that the divergence problem is avoided, and the quality of the estimate is improved. Indeed, perfect results are obtained for ideal data satisfying the underlying assumptions. Experimental results are presented to support these claims.

2. Pentland's Method

2.1. Technical Background

We start by introducing some notation. For any function f on subset Ω of the xy-plane, let $\mathbb{E}^{\Omega}\{f\}$ be the expected value of f given by

$$\mathbb{E}^{\Omega}\{f\} = \frac{1}{|\Omega|}\int_{\Omega} f(x,y)\,dxdy,$$

where $|\Omega|$ denotes the area of Ω, and let $\mathrm{Var}^{\Omega}\{f\}$ be the variance of f defined by

$$\mathrm{Var}^{\Omega}\{f\} = \sqrt{\mathbb{E}^{\Omega}\{f^2\} - \left(\mathbb{E}^{\Omega}\{f\}\right)^2}.$$

Given a vector $\mathbf{s} = (s_1, s_2)$ and (x, y) in Ω, let $f_{\mathbf{s}}(x, y)$ denote the partial derivative of f at (x, y) in direction $\mathbf{s}$ defined by

$$f_{\mathbf{s}}(x,y) = s_1 f_x(x,y) + s_2 f_y(x,y),$$

where $f_x(x, y)$ and $f_y(x, y)$ denote the partial derivatives of f at (x, y) in directions $(1, 0)$ and $(0, 1)$, respectively.

We now consider the geometry and photometry of image formation, and formulate the source estimation problem. Suppose that a Lambertian surface S with constant albedo μ is illuminated by an infinitely distant sun from the direction determined by the unit vector $\mathbf{l} = (l_1, l_2, l_3)$. Define the slant, σ_l, and tilt, τ_l, of the source vector $\mathbf{l}$ by the formula

$$\mathbf{l} = (\sin\sigma_l \cos\tau_l, \sin\sigma_l \sin\tau_l, \cos\sigma_l) \tag{1}$$

with $0 \le \sigma_l \le \pi$ and $0 \le \tau_l < 2\pi$. Assume that S is viewed from the direction determined by the vector $(0, 0, 1)$. Let $\mathbf{n}(x, y)$ be the unit normal to S such that the scalar product of $\mathbf{n}(x, y)$ and the viewing vector $(0, 0, 1)$ is non-negative at the point whose (orthographic) projection along the z-axis onto the xy-plane coincides with (x, y). Then the irradiance $E(x, y)$ of the image of S on a plane parallel to the xy-plane is given by

$$E(x,y) = \mu\mathbf{n}(x,y)\cdot\mathbf{l}. \tag{2}$$

This *image irradiance equation* holds over the image domain Ω, where $E(x, y) > 0$. The light source estimation problem may now be expressed simply as the need to determine $\mathbf{l}$, or equivalently σ_l and τ_l, given E.

Under the assumption that S is a hemisphere and $\mathbf{s}$ is a unit vector (satisfying $s_1^2+s_2^2=1$), Pentland proposed the following formulae for slant and tilt of the source direction:

$$\sigma_l = \arccos\left[1-\frac{\left(\mathbb{E}^{\Omega}\{E_x\}\right)^2+\left(\mathbb{E}^{\Omega}\{E_y\}\right)^2}{\left(\mathrm{Var}^{\Omega}\{E_{\mathbf{s}}\}\right)^2}\right]^{1/2} \tag{3}$$

and

$$\tau_l = \arctan\frac{\mathbb{E}^{\Omega}\{E_y\}}{\mathbb{E}^{\Omega}\{E_x\}}. \tag{4}$$

Here, the slant formula is tacitly assumed to be independent of any particular choice of $\mathbf{s}$. If both (3) and (4) were correct, then one might reasonably expect exact estimates of source direction given images of spheres. This begs the question of the expected accuracy of the estimates in the face of images of non-spherical surfaces. In this situation, one might expect reasonably accurate responses providing the distribution of irradiance derivatives of images of non-spherical surfaces are sufficiently close to the distribution of irradiance derivatives of images of spheres. In fact, these expectations are not met. As we show below, even when applied to an ideal image of a sphere, the formula for slant in (3) is incorrect. In contrast, the tilt formula (4) is mathematically sound and will not be discussed.

2.2. Invalidity of Slant Formula

We proceed to establish the invalidity of (3). Let S be the hemisphere given by the graph of the function

$$z(x,y)=\sqrt{R^2-x^2-y^2},$$

where R is a positive number and (x,y) runs over the disc in the xy-plane centered at the origin with radius R. Assume that S is viewed from the direction determined by the vector $(0,0,1)$. Then the corresponding unit normal $\mathbf{n}$ at a point (x,y,z) in S takes the form $(x/R, y/R, \sqrt{R^2-x^2-y^2}/R)$. Suppose that the source vector $\mathbf{l}$ is such that $l_3 \neq 0$. Now, in accordance with (2), the image of the hemisphere is given by

$$E(x,y)=\frac{\mu}{R}(l_1x+l_2y+l_3\sqrt{R^2-x^2-y^2}), \tag{5}$$

over the domain $\Omega=\{(x,y)\in\mathbb{R}^2\colon l_1x+l_2y+l_3\sqrt{R^2-x^2-y^2}>0, x^2+y^2\leq R^2\}$. The invalidity of (3) is established by proving that, for each unit vector $\mathbf{s}$,

$$\mathrm{Var}^{\Omega}\{E_{\mathbf{s}}\}=+\infty. \tag{6}$$

As the expected values $\mathbb{E}^{\Omega}\{E_x\}$ and $\mathbb{E}^{\Omega}\{E_y\}$ are finite, the last equality implies that if (3) were to hold, then the only admissible value of σ_l would be 0. Clearly, the finiteness $\mathbb{E}^{\Omega}\{E_x\}$ and $\mathbb{E}^{\Omega}\{E_y\}$ also implies that $\mathbb{E}^{\Omega}\{E_{\mathbf{s}}\}$ is finite for each unit vector $\mathbf{s}$. Thus (6) will follow once we show that $\mathbb{E}^{\Omega}\{E_{\mathbf{s}}^2\}=+\infty$ for any unit vector $\mathbf{s}$.

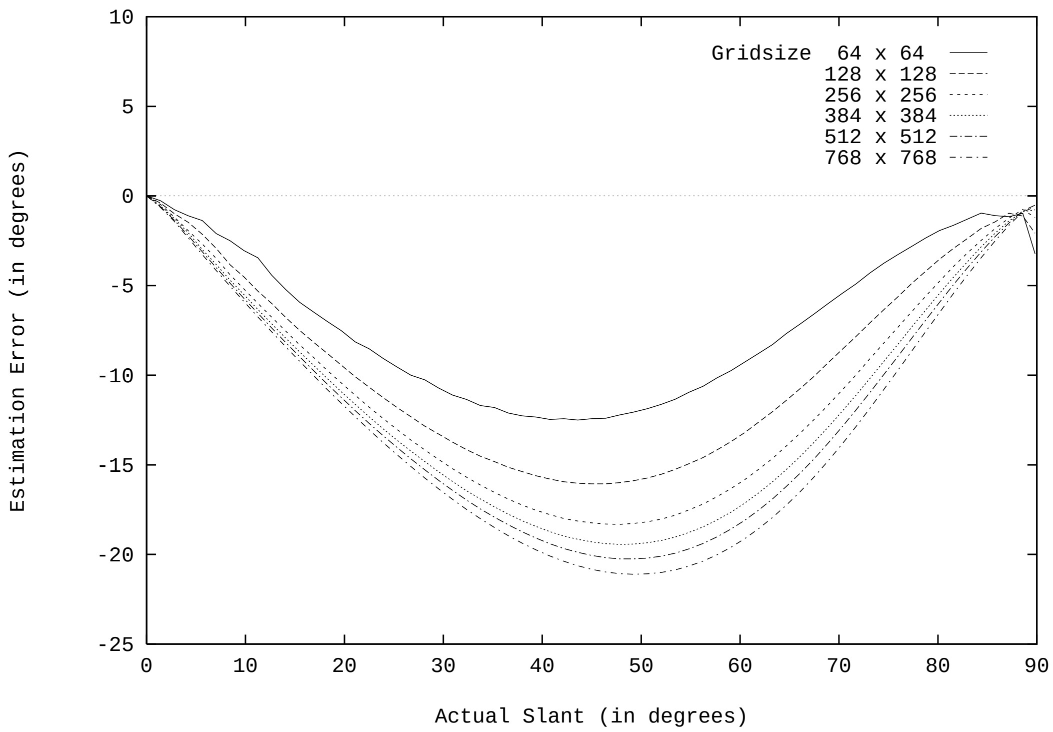

Figure 1: Slant error against actual slant for Pentland's method.

This indeed turns out to be the case, as is shown in a companion paper[1]. Pentland's formula for slant is thus shown to be invalid.

2.3. Experimental Results

In practice, if Pentland's technique is applied to an image of a sphere in such a way that image points are always confined within the boundary, then expected values for the square of image irradiance derivatives, obtained via finite differencing, will always be bounded. It might therefore seem that the above divergence problem with the slant formula has no practical impact. However, as the resolution of the image is increased (with a more dense grid of image points), then the expected value for the square of image irradiance derivatives will increase without bound. This acute sensitivity to image resolution is clearly a highly undesirable property.

Figure 1 highlights the change in performance of Pentland's slant estimator as a function of image resolution. For a range of image resolutions, we plot error in the estimated slant versus actual slant. The images used are those of a sphere illuminated from various directions. Each graph is derived from 65 slant estimates, where the actual slant ranges over 0 to 90 degrees. Performance of the estimator is worse in the mid-range, where it may clearly be seen that increased image resolution results in a poorer slant estimate. Of course, it may also be observed from Figure 1 that the Pentland method is far from exact when confronted with ideal data.

3. Disc Method

To alleviate the shortcomings of Pentland's method, we propose an estimator, that we term the disc method, which adjusts the domain of integration in the formulae for slant and tilt so that points close to the edge of the image domain are no longer involved. This adjustment ensures, among other things, that expected values of squared image derivatives remain finite. The derivation of the estimator is omitted here; for details, the interested reader is referred to Ref. 1.

Once again, assume that the image depicts the Lambertian hemisphere with constant albedo, given by the graph of the function $z(x, y) = \sqrt{R^2 - x^2 - y^2}$ for some positive R. Let $0 < \alpha < 1$ be such that the disc, D_α, centered at the origin with radius αR, is contained in the image domain. As before, let $\mathbf{s} = (s_1, s_2)$ be a unit vector. It may then be shown that the tilt and slant of source direction are given by

$$\tau_l = \arctan \frac{\mathbb{E}^{D_\alpha}\{E_y\}}{\mathbb{E}^{D_\alpha}\{E_x\}}$$

and

$$\sigma_l = \arccos \left[1 + \frac{\theta(\alpha)\Big[\big(\mathbb{E}^{D_\alpha}\{E_x\}\big)^2 + \big(\mathbb{E}^{D_\alpha}\{E_y\}\big)^2\Big]}{\big(\mathrm{Var}^{D_\alpha}\{E_{\mathbf{s}}\}\big)^2} \right]^{-1/2},$$

where

$$\theta(\alpha) = -\frac{1}{2} - \frac{1}{2\alpha^2}\ln(1 - \alpha^2) = \frac{\alpha^2}{4} + \frac{\alpha^4}{6} + \frac{\alpha^6}{8} + \cdots.$$

As we see, Pentland's formula for tilt also applies when the domain of integration is confined to a disc that is smaller than the entire image domain. A similarly confined domain is also used in the slant estimate; however, Pentland's formula for slant this time undergoes significant change.

Figure 2 shows the disc method's slant estimator at work. To enable comparison, the graphs in Figures 1 and 2 are similarly scaled. We verify experimentally that, for an image of a sphere, estimated slant errors are very small and are not significantly affected by increased resolution. It should be stressed that the disc method has been developed simply to highlight and overcome the deficiencies of the Pentland method. We do not claim that the disc method is especially suitable for widespread utility. An analysis of the performance of the disc method may be found in Ref. 2.

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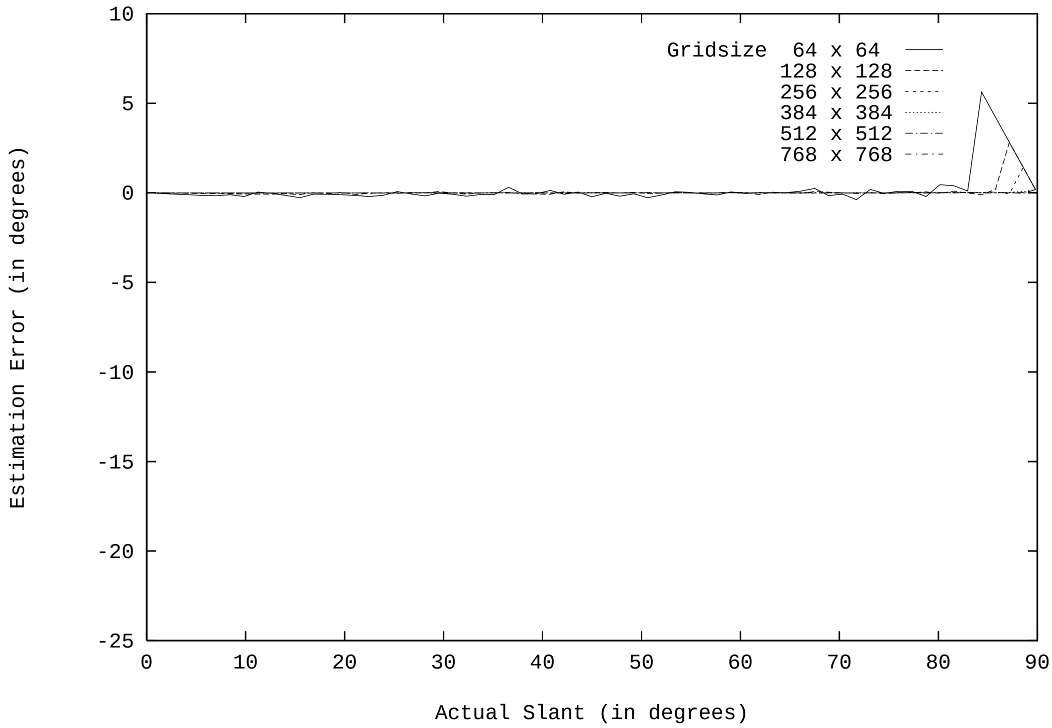

Figure 2: Slant error against actual slant for the disc method.

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